

Mathematical Modeling of Social Phenomena

Evolutionary Game Theory

Lecture layout

Game theory repetition

ESS

Replicator dynamics

Discussing your final papers

What is a game?

A set of players:

$$P = \{p_1, \dots, p_k\}$$

A set of strategies:

$$S = \{s_1, \dots, s_n\}$$

A mapping from a tuple of selected strategies to a payoff:

$$\square : S \times \dots \times S \rightarrow \mathbf{R}$$

Payoff versus utility

The payoff function:

$$\square : S \times \dots \times S \rightarrow \mathbf{R}$$

But what is the utility function?

Altruism?

Morality?

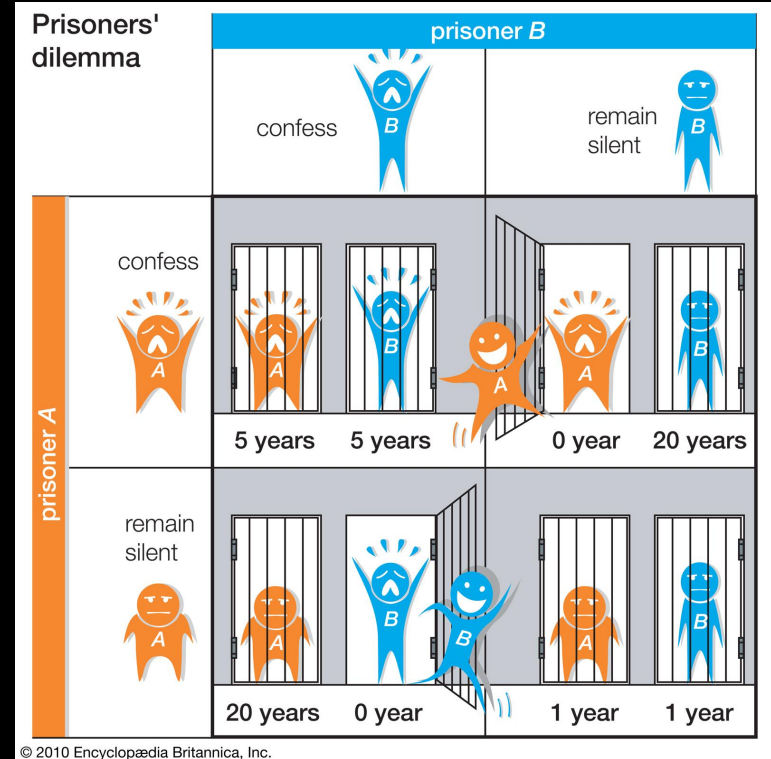
How does the book chapter handle this?

On dominant strategies

A strategy tuple (s_1^*, s_2^*) in $S \times S$ is a Nash equilibrium if for all players i and strategies s_i ,

$$u_i(s_1^*, s_2^*) \geq u_i(s_i, s_2^*)$$









I.e., no one has an incentive to independently change their strategy.



Risk- v. payoff dominant

Harder to coordinate,
because we don't know if
the others are either or?









Risk dominance tend to
win out.

 S _i h	 COOPERATE DEFECT	
COOPERATE 		
DEFECT 		

Anti-coordination game

Pre-commitment

Hawk-Dove Model: Costs and Benefits of Fighting over Resources

Payoff* to...	...in fights against:	
	 hawk	 dove
 hawk	Hawk wins 50% of fights; is injured in 50% of fights.  Payoff: $(V-D)/2$	Hawk always wins; dove flees.  Payoff: V
 dove	Dove never wins; is never injured.  Payoff: 0	Dove wins 50% of fights; is never injured; wastes time.  Payoff: $V/2 - T$

* V = fitness value of winning resources in fight

D = fitness costs of injury

T = fitness costs of wasting time

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Mixed strategy

2 0

0 1

What are the three equilibria?

Optimality

Individual

Pareto

A choice of strategies—one by each player—is Pareto-optimal if there is no other choice of strategies in which all players receive payoffs at least as high, and at least one player receives a strictly higher payoff.

Social

Evolutionary game theory

Or rather, ESS

Fitness as a result of interaction

Beatles size

Game:

5,5 1,8

8,1 3,3

s_1 is an ESS if there is a positive number y such that if any other strategy invades at a level $x < y$, the payoff of s_1 is greater than of the invader

Evolutionary arms races

What are they?

A sign of an intelligent designer?

The ESS conditions

Game: a, a b, c
 c, b d, d

In a two-player, two-strategy, symmetric game, s_1 is evolutionary stable precisely when either

$$a > c$$

or

$$a = c, \text{ and } b > d.$$

Nash equilibria and ESS

Why is an ESS a Nash equilibria?

In game: 1, 1 0, 0
 0, 0 2, 2

The three equilibria, are they Nash, are they ESS?

ESSs in the Hawk and dove game

	Dove	Hawk
Dove:	3, 3	1, 5
Hawk	5, 1	0, 0

$$V(p, q) = 3pq + p(1-q) + 5(1-p)q + 0(1-q)(1-p)$$

ExPayoff of Dove: $3p + 1-p = 1+2p$

ExPayoff of Hawk: $5p$.

Implies $p = \frac{1}{3}$.

$V(p, p) = V(q, p)$ for all q , due to the indifference

$\Rightarrow V(p, q) > V(q, q)$?

Actually just go through it as it stands on page 224.

Replicator dynamics